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Competition vs. quality in an industry with imperfect traceability.

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Abstract

We consider an industry where firms produce goods that have different quality levels but firms cannot differentiate themselves from rivals. In this situation, producing low-quality generates a negative externality on the whole industry. This is particularly true when consumers cannot identify producers. In this article, we show that under a "Laissez Faire" situation free entry is not socially optimal and we argue that the imposition of a Minimum Quality Standard (MQS) may induce firms to enter the market.

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1 Introduction

There exist situations where products are not traceable by consumers and consumers are not able to identify either the producer or the level of quality of products or services. When doing their choices, consumers mainly base their decisions on the reputation of the entire industry. In this sense, firms share, at least partially, the reputation of the industry. An empirical evidence for this phenomena is food safety. Food safety is a credence attribute of product: consumers are not able, because it is too costly, to check the real quality of the product even after consumption. Even if products may have different safety levels, consumers consider products as generic (e.g fresh produce). Indeed, after an outbreak of food poisoning, everyone in the industry will suffer from the safety outbreak. Since mid-May, the “E-coli cucumber outbreak” has killed 16 people and infects more that 1100 people in Europe. The Spanish Federation of Producers / exporters (FEPEX) estimates lost sales up to €200 million per week. The cucumber crisis has also affected French producers, who, according to the tomatoes and cucumbers producers association suffer from a fall of sales of French cucumbers by 75%.1

In this article, we address the issue of entry in an industry where firms produce different quality levels but cannot differentiate themselves from their rivals. Also, producing low-quality generates a negative externality on the whole industry. We build a simple model and we show that the link between market structure and welfare is ambiguous. In the “Laissez Faire” situation, an increase in the number of firms has two opposite effects. First, it leads the price to decrease increasing welfare. Second, incentives to free ride increase, reducing the average level of quality and then reducing welfare. Free entry is thus not socially optimal. Contrarily to conventional wisdom, we argue that the imposition of a Minimum Quality Standard may induce firms to enter the market and increase welfare.

We have in mind the safety issue of fresh fruit and vegetables in Europe which is one-dimensional, as opposed to the United States where regulation on the safety of those produce also refers to the presence of microbiological hazards such as E-coli, Salmonella, etc. The definition of food safety for fresh fruit and vegetables in Europe relies on the Maximum Residue Limits for pesticides (MRLs) set by the European authorities (Regulation (EC) No 396/2005). Residues found in or on produce are judged, according to these laws, as being above, at or below the limit. Any food operator must comply with a “performance standard”, as defined in Henson and Caswell (1999): the food product they market should reach the prescribed product quality standards and/or safety levels. How they do reach the standard is left to the discretion of the food operators. Public agencies in charge of enforcing law and monitoring food safety, mostly conduct regular on-site and

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1Nouvel observateur: Economy Real Time
product-oriented inspections. In the case of fresh produce, samples are collected and laboratory analyses are carried out to check that residue levels are within the legal limits (see for instance Rouvière et al. 2010). If excess levels are found, food operators are found guilty of an offence and the whole box of the incriminated product is taken off the market. The cost of conducting a laboratory analysis (€300 in average) prevents from analyses conducted by consumers.

The closest literature on this issue is the literature about collective reputation. Tirole (1996) considers that collective reputation should be assumed to be the aggregate reputation of individual agents. In a context of imperfect information available to consumers about quality, he shows that the composition of the producer group matters. Winfree and McCluskey (2005) assume that collective reputation is a common property resource and show that the (exogenous) number of firms should be considered closely because of free-rider effects. However, in those studies, the size of the group of producers is taken as fixed and then does not allow for entry in or exit from the group. Our model, although static, endogeneizes the entry decision.

Moreover, our article directly participates to the controversial debate in the industrial organisation literature as regards to the effect of a Minimum Quality Standard (MQS) on competition (for instance, see Leland 1979). Ronnen (1991) shows that an adequate MQS can increase both quantities sold and quality and then social welfare. The intuition of this result is that an increase in the low quality induces an increase of the high quality (in order to soften price competition) but equilibrium prices are however lower and more consumers buy the product (see also Crampes and Hollander 1995 for a similar result). The robustness of this result has been questioned in few direction. Valetti (2000) shows that this statement is sensitive to the mode of competition and Scarpa (1998) shows that it depends on the duopolistic market structure. Garella and Petrakis (2008) justify the use of MQS in industries where consumers face imperfect information. They point out that a MQS will change the consumers’ perception on quality. However, none of these papers consider the possibility of entry and/or exit. As Boccard and Wauthy (2005) have already underlined, studying quality regulation through quantity regulation, MQS would induce firm to exit the market and/or reduce the entry of new firms. Our model of quality differs from previous studies because there is no differentiation but quality externalities.

The article proceeds as follows. We set up the theoretical model to emphasize the free entry issue in a “Laissez Faire” situation. Next, we analyse the competition effect when a MQS is imposed on the industry. Finally, we provide our conclusions and their policy implications.
2 The Model

We focus on an industry in which identical and risk neutral firms choose their level of quality in order to avoid quality failures. Products may have different quality levels but quality is a credence attribute: consumers are not able to observe these different quality levels even after consumption. Then, consumers only rely on the reputation of the entire industry.

Because we consider a static framework where the reputation of the industry depends on the produced levels of quality, we don’t rely on the past history of the industry. The model can thus be interpreted as an investigation of the reputational problem faced by an infant industry.

We consider a two-stage game. In the first stage, profit maximising firms choose whether or not to enter the market. If a firm enters the market, it faces a fixed (sunk) cost $F > 0$. Since we focus on quality, each firm produces one unit of the product. In the second stage, the firm chooses a quality level $s_i \geq 0$ with cost $C(s_i)$ where $C' > 0$ and $C'' > 0$. We assume that the reputation of the industry is “good” with a probability $R(s_a)$ that only depends on the average level of quality (for simplicity) which is given by $s_a$ with

$$s_a = \frac{\sum_{i \in N} s_i}{n} \quad (1)$$

where $N$ denotes the set of the $n$ firms which enter the market, with $R' > 0$ and $R'' \leq 0$. The industry reputation is “bad” with probability $1 - R(s_a)$. The inverse demand function is then $P(n)$ (with $P' < 0$) if the reputation of the industry is “good”, and the inverse demand function is 0 if the reputation of the industry is “bad”\(^2\). Therefore, the expected profit of firm $i$ is

$$\Pi_i = R(s_a) \cdot P(n) - C(s_i) - F \quad (2)$$

We make the following assumptions on the profit function which hold all through the paper.

**Assumption 1:** The profit of a monopolistic firm is non negative when its quality level is optimal,

$$F \leq R(s_M) \cdot P(1) - C(s_M) \, , \quad (3)$$

\(^2\)This is simply a normalisation. Indeed, suppose that if the reputation is "bad", the inverse demand drops to $\alpha P(n)$ with $0 \leq \alpha < 1$. The expected inverse demand is $R(s_a) \cdot P(n) + (1 - R(s_a)) \cdot \alpha P(n)$. It can be rewritten as $(R(s_a) + (1 - R(s_a)) \cdot \alpha) P(n)$. To see that our assumption is a normalisation, simply relabel $(R(s_a) + (1 - R(s_a)) \cdot \alpha)$ as $R(s_a)$.\]
where $s_M$ denotes the optimal quality effort of the monopolistic firm, i.e.

$$s_M = \arg \max \{ R(s) P_M - C(s), \ s \geq 0 \}. \quad (4)$$

**Assumption 2:** A monopolistic firm’s profit is non positive when its quality level is large enough,

$$\lim_{s \to +\infty} (R(s) P(1) - C(s) - F) \leq 0. \quad (5)$$

In our setting, we can apply the result provided in Spence (1975) (see proposition). The (expected) inverse demand function is $R(s)P(n)$ and we have

$$\frac{\partial^2 (R(s)P(n))}{\partial s \partial n} < 0, \quad (6)$$

thus the monopoly always undersupplies quality (as with consumers’ preferences à la Mussa-Rosen (Mussa and Rosen, 1978)). In other words, the monopoly will always underinvest in quality because of the distortions that exist between firms and the society. Our model differs from those seminal works since we focus on an oligopoly where quality externality among firms induces a second and new distortion.

### 3 “Laissez faire” situation

In this section, we solve the game described above where there is no intervention from the regulator. We solve the game through backward induction.

First, we solve the second stage of the game. Assume that $n$ identical firms entered the market in the first stage. Firms individually make their quality choice, $s_i$. The optimisation problem for firm $i$ is then

$$\max_{s_i \geq 0} (R(s_a) P(n) - C(s_i)), \quad (7)$$

The first order condition is

$$\frac{1}{n} R'(s_a) P(n) = C'(s_i). \quad (8)$$

This condition allows to define firm $i$’s best response as an implicit function of the average quality $s_a$ (and of the number of firms $n$) as usual in “private provision of a public good” games. Note that $\frac{\partial s_i}{\partial s_a} = \frac{\frac{1}{n} R'(s_a) P(n)}{C'(s_i)} \leq 0$. Hence, as the average quality $s_a$ increases, firm $i$ has an incentive to decrease its quality level.

In an interior equilibrium, the firms’ quality levels are identical (due to the convex nature of the cost function $C$), i.e. for all $i$, $s_i^* = s^*$ which is characterised
This equilibrium condition implicitly defines the equilibrium quality level, \( s^* \), as a function of the number of firms \( n \).

**Proposition 1** An increase in the number of firms lowers the equilibrium quality level, \( \frac{ds^*}{dn} < 0 \).

When the number of firms increases firms have incentives to decrease their quality level. First, quality efforts are diluted in the industry reputation then firms' incentives to free ride increase (this results is similar to Winfree and McCluskey (2005)). Second, the price of the product decreases. Each firm’s expected benefits decrease then firms provide a lower quality level.

Second, we derive the subgame perfect equilibrium of the game. In the first stage, firms anticipate the equilibrium quality level (characterised at stage 2) and decide to enter the market if their ex-ante expected profit is non negative. The number of firms who enter the market \( n^* \) is then characterised by:

\[
R \left( s^* \left( n^* \right) \right) P \left( n^* \right) - C \left( s^* \left( n^* \right) \right) = F;
\]

(10)

where \( n^* \) (\( \geq 1 \) according to Assumption 1) denotes the equilibrium number of firms which is an implicit function of \( F \), the sunk cost of entry. Differentiating condition (10) with respect to \( F \) we obtain:

\[
\frac{dn^*}{dF} = \left[ R' \left( s^* \right) P \left( n^* \right) - C' \left( s^* \right) \right] \frac{ds^*}{dn} + R \left( s^* \right) P' \left( n^* \right) \right]^{-1}.
\]

(11)

From condition (9), we obtain

\[
R' \left( s^* \right) P \left( n^* \right) - C' \left( s^* \right) = \left( n^* - 1 \right) C' \left( s^* \right) \geq 0
\]

(12)

When a firm decides to enter the market, it anticipates that the price \( P' \left( n^* \right) < 0 \) and the equilibrium quality will decrease \( \frac{ds^*}{dn} < 0 \). Consequently, the number of firms increases only if the entry cost decreases:

\[
\frac{dn^*}{dF} < 0.
\]

(13)

This result strongly depends on the fact that the number of firms has a negative impact on the equilibrium quality.

**Welfare effect of the market structure:** In order to appraise the welfare effect, we consider the equilibrium quality game (stage 2), where each firm provides the
same (second stage equilibrium) quality level \( s^*(n) \) defined by condition (9), with \( 1 \leq n \leq n^* \). We focus on the effect of an increase in the number of firms on consumer surplus and on social welfare.

**Consumer Surplus:** Under the assumption of quasi-linear consumer utility, when there are \( n \) firms, the expected (Marshalian) consumer surplus is

\[
CS(s^*, n) = R(s^*) \left[ \int_0^n P(z) \, dz - P(n) \, n \right].
\]

(14)

The marginal effect of an increase in the number of firms on the expected consumer surplus is

\[
\frac{dCS}{dn} = \frac{\partial CS}{\partial n} + \frac{\partial CS}{\partial s^*} \frac{ds^*}{dn}.
\]

(15)

The direct effect is given by

\[
\frac{\partial CS}{\partial n} = R(s^*) \left[ -P'(n) \, n \right] > 0,
\]

(16)

i.e. consumer surplus increases through a decrease in the price of the product. The indirect effect, \( \frac{\partial CS}{\partial s^*} \frac{ds^*}{dn} \), represents the effect of an increase in the number of firms through its impact on the equilibrium quality. We know from Proposition 1 that \( \frac{ds^*}{dn} < 0 \). The effect of an increase of the quality level on consumer surplus is given by

\[
\frac{\partial CS}{\partial s^*} = R'(s^*) \left[ \int_0^n P(z) \, dz - P(n) \, n \right] > 0.
\]

(17)

Then, \( \frac{\partial CS}{\partial s^*} \frac{ds^*}{dn} < 0 \), i.e. the indirect effect is negative. Finally, the global effect of an increase of the number of firms on consumer surplus is ambiguous because both the price and the quality of the product decrease.

**Social Welfare:** Social welfare is denoted by \( W = W(s^*, n) \), with \( W(s^*, n) \) given by:

\[
W(s^*, n) = R(s^*) \int_0^n P(z) \, dz - n \left[ C(s^*) + F \right],
\]

(18)

We now evaluate the welfare effect of competition. Differentiating condition (18) with respect to \( n \), we obtain

\[
\frac{dW}{dn} = \frac{\partial W}{\partial n} + \frac{\partial W}{\partial s^*} \frac{ds^*}{dn}.
\]

(19)
The welfare effect is twofold. The direct effect is given by
\begin{equation}
\frac{\partial W}{\partial n} = R(s^*)P(n) - [C(s^*) + F].
\tag{20}
\end{equation}

As long as profits remain non-negative, \( \frac{\partial W}{\partial n} \) has a positive value. This represents the classical positive effect of competition. The indirect effect is given by \( \frac{\partial W}{\partial s^*} \). According to Proposition 1, the quality level decreases with respect to the number of firms, \( \frac{ds^*}{dn} < 0 \).

The welfare effect of an increase in the quality level is given by
\begin{equation}
\frac{\partial W}{\partial s^*} = R'(s^*) \int_0^{n^*} P(z) \, dz - n^*C'(s^*). \tag{21}
\end{equation}

Since \( P' < 0 \), we have
\begin{equation}
P(n^*) < \int_0^{n^*} P(z) \, dz. \tag{22}
\end{equation}

According to the latter condition and (9) \( \frac{\partial W}{\partial s^*} \) has a positive value. Therefore, the indirect welfare effect, \( \frac{\partial W}{\partial s^*} \frac{ds^*}{dn} \), has a negative value. The welfare effect of competition is ambiguous. An increase in the number of firms reduces each firm’s market power and prices, thereby improving social welfare. Yet at the same time, it lowers the average quality, reducing social welfare.

**Proposition 2** Under the “Laissez Faire” situation, at the free entry point the number of firms is larger than the optimal number of firms.\(^3\)

Proposition 2 states that \( n^* > n^W \),\(^4\) where \( n^W \) represents the number of firms that maximizes social welfare. Figure 1 illustrates this result.\(^5\)

\(^3\)Until now we have ignored the integer problem. Our results are qualitatively unaffected if we consider \( n \) as an integer (the mathematical writing is a bit different).

\(^4\)Considering \( n \) has an integer, this inequality would be weak. Indeed, in the space of real numbers, the welfare function may have an optimum reached between two integers and the free entry point may also be between these two integers. In this case, the welfare optimal integer level is the free entry integer number of firms.

\(^5\)Figure 1 represents the following specification of the model. The industry reputation is characterized by a logit function of the average quality, \( s_a \): \( R(s_a) = \frac{e^{sa}}{1+e^{sa}} \). The inverse demand function is assumed to be linear, \( P(n) = \alpha - n \) where \( \alpha > 1 \). The cost function is \( C(s_i) = \frac{1}{2} (1 + s_i)^2 \).
Figure 1 also illustrates the welfare effects of competition. Welfare first increases and then decreases with the number of firms. When \( n^* \) firms compete in the market under the “Laissez Faire” situation, the positive welfare effect of competition is lower than the negative effect of free-riding on quality. Therefore, the regulator needs to intervene in order to avoid free-riding incentives and to prevent the entire industry from failing to perform. This result contributes to the critical debate in the industrial organisation literature that concerns the justification of anti-competitive regulation. For instance, Mankiw and Whinston (1986) have shown that in homogeneous product markets, free entry can lead to a socially excessive number of firms. They model a situation in which the output per firm falls as the number of firms in the industry increases. In our model, we assume that the output per firm is constant, however, the free-riding incentives lead us to the same conclusion.

4 Minimum Quality Standard

In this section, while maintaining our focus on the entry issue, we examine the situation where the regulator imposes a Minimum Quality Standard (MQS). We assume that, before stage 1, a MQS \( s \) is announced. Firms decide to enter the market at stage 1 and choose a quality level \( s_i \geq s \) at stage 2. Since the purpose of this section is to compare the effect of different levels of MQS, we do not consider the regulator as a player, that is \( s \) is given.

**Market structure and MQS:** In this section, we derive the equilibrium of the game for different levels of the MQS, \( s \geq 0 \). The equilibrium quality and the equilibrium number of firms will depend on the level of the MQS. Let us denote \( s^{**} = s^{**}(s, n) \) the equilibrium quality of stage 2 and \( n^{**} = n^{**}(s, F) \) the equilibrium number of firms. In the previous section, we have characterised the
equilibrium of this game under the “Laissez Faire” situation, that is for $s = 0$. In other words, $s^* (0, n)$ and $n^* (0, F)$ are such that $s^* (0, n) = s^* (n)$ and $n^* (0, F) = n^* (F)$, where $s^*$ characterised by condition (9) and $n^*$ characterised by condition (10).

In order to present the next proposition, we need to define a particular quality level and a particular number of firms denoted by $s_c$ and $n_c$, respectively. $s_c$ and $n_c$ are defined as the equilibrium quality level and the equilibrium number of firms of the following two stage game: at stage 1, firms enter the market if their expected profit is non negative, and at stage 2 firms behave cooperatively, i.e. each firm provides the same quality level in order to maximise the total profit of the industry, $n (R (s) P (n) - C (s))$. $s_c$ and $n_c$ are characterised by $R' (s_c) P (n_c) = C' (s_c)$ and $R (s_c) P (n_c) - C (s_c) - F = 0$.

**Proposition 3** There exists a unique symmetric equilibrium and,

(i) If $\underline{s} \leq s^*$, then the MQS has no effect on quality, i.e. $s^{**} = s^*$, neither on competition, i.e. $n^{**} = n^*$,

(ii) If $s^* < \underline{s}$, then the MQS is binding $s^{**} = \underline{s}$. There exists $s' \geq s_c$ such that for $s^* \leq \underline{s} \leq s'$, $n^{**} \geq n^*$ and for $s' < \underline{s}$, $n^{**} < n^*$. The maximal number of firms is $n_c$ and is achieved for $\underline{s} = s_c$.

Relatively to the “Laissez Faire” situation: If the MQS is not binding ($\underline{s} \leq s^*$), the MQS does not alter either competition or the firm’s quality level.

We discuss now the case when the MQS is binding: Increasing the level of the MQS ($s^* < \underline{s} < s_c$) increases the level of the industry reputation by increasing firms’ quality levels. The MQS induces firms to enter the market as long as the cost of providing the MQS level is sufficiently low. When the MQS equals to the cooperative equilibrium quality level ($\underline{s} = s_c$), the industry reputation is maximal. When the MQS is imposed at such a level, a maximum number of firms ($n_c$) enters the market. For MQS levels which are higher than the cooperative equilibrium quality level ($\underline{s} > s_c$), the marginal cost of providing quality overcomes the marginal benefit that leads to a drop in profits. However, the number of firms remains higher than it would be under the “Laissez Faire” situation as long as the MQS is low enough ($s_c < \underline{s} \leq s'$). For the highest MQS levels ($s' > \underline{s}$), the number of firms becomes lower than the number of firms in the “Laissez Faire” situation ($n^*$). This is the only situation in which the MQS can reduce competition. Figure 2 illustrates those results.
In the light of these statements, we turn now to analyse the welfare effect after the introduction of a given MQS.

**Welfare effect of the MQS:** When a MQS $s$ is imposed, the social welfare function can be written as

$$W(s^{**}, n^{**}) = R(s^{**}) \left[ \int_{0}^{n^{**}} P(z)dz - n^{**}P(n^{**}) \right].$$

(23)

According to the result of Proposition 3, we can provide the following relationship between the level of the MQS and social welfare:

**Corollary 4** Relatively to the “Laissez Faire” situation, social welfare is (i) unaffected when the level of the MQS is sufficiently low ($s \leq s^*$), (ii) improved when the level of the MQS is in a middle range ($s^* < s \leq s'$).

Relatively to the “Laissez Faire” situation, the introduction of a MQS unambiguously improves welfare as long as the level of the MQS leads to a greater number of active firms.

## 5 Conclusion

We have considered industries where firms provide different quality levels. They cannot differentiate themselves from their rivals but can suffer from externalities due to rivals low-quality levels. We have shown that a “Laissez Faire” situation leads to a sub-optimal number of firms in the market. The regulator face different solutions which all have their positive and negative effects both on quality and competition. In such a case, the regulator face a trade-off between quality and
competition. The regulator can choose to restrict the number of firms in the market. On the one hand, such regulation would limit the incentive to free ride and then provide a sufficient level of quality. On the other hand, this regulation has also two negative effects. First, it leads to an increase in the price. Second, free riding incentives are reduced but they are not eradicated. The other solution available is the introduction of a Minimum Quality Standard. We have shown that a Minimum Quality Standard can eradicate incentives to free-ride and can sustain both a high average level of quality and a high degree of competition.
Appendix

Proof of Proposition 1

Differentiating condition (9) with respect to \( n \) we obtain

\[
\frac{ds^*}{dn} = \left[ -\frac{1}{n} P'(n) + \frac{1}{n^2} P(n) \right] \frac{R'(s^*)}{\frac{1}{n} R''(s^*) P(n) - C''(s^*)},
\]

(24)

Since \( P' < 0 \), we have \( 0 < \left[ -\frac{1}{n} P'(n) + \frac{1}{n^2} P(n) \right] \). Moreover, \( R'(s^*) > 0 \), then,

\[
\text{sign} \left[ \frac{ds^*}{dn} \right] = \text{sign} \left[ \frac{1}{n} R''(s^*) P(n) - C''(s^*) \right] < 0.
\]

(25)

Proof of Proposition 2

We evaluate the marginal variation of welfare at the free entry point. Differentiating condition (18) with respect to the number of firms \( n \), we obtain

\[
\frac{dW}{dn} (s^*, n^*) = \left[ R'(s^*) \int_0^{n^*} P(z) \, dz - n^* C'(s^*) \right] \frac{\partial s^*}{\partial n}
\]

(26)

According to Proposition 1 and

\[
\frac{\partial W}{\partial s^*} = R'(s^*) \int_0^{n^*} P(z) \, dz - n^* C'(s^*) > 0,
\]

(27)

This expression has a strict negative value.

Proof of Proposition 3

When the MQS is not binding, i.e \( s \leq s^* \), it is straightforward that \( s^{**} = s^* \) and \( n^{**} = n^* \).

When the MQS is such that \( s > s^* \), it is straightforward that \( s^{**} = s \). Considering the number of firms which enter the market at stage 1, \( n^{**} \), is characterised by

\[
R(s) P(n^{**}) - C(s) = F,
\]

(28)
Differentiating this condition with respect to $s$ leads to

$$\frac{\partial n^{**}}{\partial s} = \frac{R'(s) P(n^{**}) - C'(s)}{-R(s) P'(n^{**})}, \quad (29)$$

Then,

$$\text{sign} \left[ \frac{\partial n^{**}}{\partial s} \right] = \text{sign} \left[ R'(s) P(n^{**}) - C'(s) \right]. \quad (30)$$

$R(s) P(n^{**}) - C(s)$ is the per firm profit when all the quality levels are $s$. Per firm profit is increasing for $s \leq s_c$ and decreasing for $s_c \leq s$. Hence, $\frac{\partial n^{**}}{\partial s} \geq 0$ when $s \leq s_c$ and $\frac{\partial n^{**}}{\partial s} \leq 0$ when $s_c \leq s$. Then, $n^{**}$ achieves its maximum, $n_c$ for $s = s_c$. Moreover, according to Assumption 2, $\lim_{s \to +\infty} (R(s) P(1) - C(s) - F) \leq 0$, then $\lim_{s \to +\infty} (n^{**}) \leq 1$. Therefore, there exists $s' \geq s_c$ such that for $s^* \leq s \leq s'$, $n^{**} \geq n^*$ and for $s' < s$, $n^{**} < n^*$.

**Proof of Corrolary 4**

When the MQS is low, i.e. $s \leq s^*$, according to Proposition 3, social welfare is $W(s^{**}, n^{**}) = W(s^*, n^*)$. When the MQS is in a middle range, $s^* < s \leq s'$, according to Proposition 3, social welfare is $W(s^{**}, n^{**}) = W(s, n^{**}(s, F))$ with $s > s^*$ and $n^{**} > n^*$. Since social welfare unambiguously increases with respect to $s^{**}$ and $n^{**}$, $W(s^{**}, n^{**}) > W(s^*, n^*)$. 
References


